

RULES OF EXPONENTS

The term 2^6 means $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$.
2 is the base and 6 is the exponent.

Using the exponent key on a calculator, press **2** **y^x** **6** **=**

The display is **64**

The following are the rules for simplifying expressions with exponents.

Multiplication

Rule for Multiplication

$$a^m \times a^n = a^{m+n}$$

Example 1. Multiply. $t^3 \times t^2$

Solution: Not using the rule,

$$\begin{aligned} t^3 \times t^2 &= (t \times t \times t) \times (t \times t) \\ &= t^5 \end{aligned}$$

Using the rule,

$$\begin{aligned} t^3 \times t^2 &= t^{3+2} \\ &= t^5 \end{aligned}$$

Division

Rule for Division

$$a^m \div a^n = a^{m-n}$$

Example 2. Divide. $y^5 \div y^3$, $y \neq 0$

Solution: Not using the rule,

$$\begin{aligned} \frac{y^5}{y^3} &= \frac{y \times y \times y \times y \times y}{y \times y \times y} \\ &= y^2 \end{aligned}$$

Using the rule,

$$\begin{aligned} \frac{y^5}{y^3} &= y^{5-3} \\ &= y^2 \end{aligned}$$

Powers

Rule for Powers

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Example 3. Simplify. $(z^2)^3$

Solution: Not using the rule,

$$\begin{aligned} (z^2)^3 &= z^2 \times z^2 \times z^2 \\ &= z^6 \end{aligned}$$

Using the rule,

$$\begin{aligned} (z^2)^3 &= z^{2 \times 3} \\ &= z^6 \end{aligned}$$

Power of a Product

Power of a Product

$$(a^m b^n)^p = a^{m \times p} b^{n \times p} = a^m b^{np}$$

Example 4. Simplify. $(-2s^{5t^4})^3$

Solution:

Not using the rule,

$$\begin{aligned} (-2s^{5t^4})^3 &= (-2s^{5t^4})(-2s^{5t^4})(-2s^{5t^4}) \\ &= -8s^{15t^{12}} \end{aligned}$$

Using the rule,

$$\begin{aligned} (-2s^{5t^4})^3 &= (-2)^{1 \times 3} s^{5 \times 3 t^4 \times 3} \\ &= (-2)^3 s^{15t^{12}} \\ &= -8s^{15t^{12}} \end{aligned}$$

Power of a Quotient
$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{m \times p}}{b^{n \times p}} = \frac{a^{mp}}{b^{np}}, \quad b \neq 0$

Example 5. Simplify. $\left(\frac{3r^2}{s^3}\right)^4$

Solution:

Not using the rule,

$$\begin{aligned} \left(\frac{3r^2}{s^3}\right)^4 &= \frac{3r^2}{s^3} \times \frac{3r^2}{s^3} \times \frac{3r^2}{s^3} \times \frac{3r^2}{s^3} \\ &= \frac{81r^8}{s^{12}} \end{aligned}$$

Using the rule,

$$\begin{aligned} \left(\frac{3r^2}{s^3}\right)^4 &= \frac{3^{1 \times 4} r^{2 \times 4}}{s^{3 \times 4}} \\ &= \frac{81r^8}{s^{12}} \end{aligned}$$

EXERCISE

A

1. Simplify.

- (a) 3^4 (b) 2^5 (c) 5^2
 (d) $(-2)^3$ (e) $(-1)^7$ (f) $2^2 \times 3^2$
 (g) $(3 \times 2)^2$ (h) $(-4)^2$ (i) -4^2

2. Simplify.

- (a) $x^4 \times x^7$ (b) $b^8 \div b^7$ (c) $(m^3)^2$
 (d) $s^3 \times s^4$ (e) $t^9 \div t^4$ (f) $(r^2)^5$
 (g) $b^3 \times b^5 \times b$ (h) $(m^4)^5$
 (i) $r^{11} \div r^9$ (j) $(a^2b^3)^2$
 (k) $\left(\frac{a^6}{b^7}\right)^2$ (l) $\left(\frac{m^3}{n^2}\right)^3$

B

3. Simplify.

- (a) $3m^3 \times 4m^5$ (b) $(7x^2)(-3x^4)$
 (c) $(-2t^3)(-3t)(4t^2)$ (d) $7a^2 \times 8a^2 \times a$
 (e) $(-24b^4) \div (6b^2)$ (f) $(18s^5) \div (-9s)$
 (g) $(6x^2y^3)^2$ (h) $(2a^2bc^4)^3$
 (i) $\left(\frac{4m^2}{3n^3}\right)^2$ (j) $\left(\frac{2x^2y}{3z^3}\right)^3$

4. Simplify.

- (a) $(4x^2y^3)(5x^4y)$
 (b) $6m^3n^4 \times 3mn^3$
 (c) $(36a^3b^4) \div (9a^2b^2)$
 (d) $(50x^3yz) \div (-10xy)$
 (e) $(-6x^2y^4) \times (7x^3y^2)$

5. Simplify.

- (a) $\frac{(2a^3b^2)(-3a^4b^3)}{6ab}$
 (b) $\frac{(-3r^2s^2t^4)^3}{9r^2s}$
 (c) $\frac{(-6x^2yz^3)(-3xyz)}{2x^2yz}$
 (d) $\frac{(-2a^2b^2)^2(-3ab)}{(-6a^2b^2)}$

C

6. Simplify.

- (a) $3^a \times 3^b \times 3^c$ (b) $2^{x+y} \div 2^{x-y}$

Fractional Exponents

For the multiplication rule to be true for fractional exponents, we must define $4^{\frac{1}{2}}$ so that

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^{\frac{1}{2} + \frac{1}{2}} = 4$$

but $\sqrt{4} \times \sqrt{4} = 2 \times 2 = 4$

Therefore $4^{\frac{1}{2}} = \sqrt{4}$

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$

Further $a^{\frac{m}{n}} \begin{cases} (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m \\ (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} \end{cases}$

Example 3. Simplify.

- (a) $36^{\frac{1}{2}}$ (b) $27^{-\frac{1}{3}}$ (c) $16^{\frac{3}{4}}$

Solution:

$(a) \ 36^{\frac{1}{2}} = \sqrt{36} = 6$	$(b) \ 27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$	$(c) \ 16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = (2)^3 = 8$
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B

1. Simplify.

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|------------------------|---------------------|-----------------------------|
| (a) 7^0 | (b) $4^0 + 5^0$ | (c) 2^{-2} |
| (d) $\frac{1}{4^{-2}}$ | (e) $\frac{1}{3^0}$ | (f) $\frac{3^{-2}}{2^{-3}}$ |
| (g) 10^{-4} | (h) $(-5)^0$ | (i) -9^0 |
| (j) $(-2)^3$ | (k) $(-3)^{-2}$ | (l) $(-1)^{-5}$ |

2. Simplify.

- | | | |
|-------------------------|-------------------------|--------------------------|
| (a) $9^{\frac{1}{2}}$ | (b) $16^{\frac{1}{2}}$ | (c) $8^{\frac{1}{3}}$ |
| (d) $16^{\frac{1}{4}}$ | (e) $8^{\frac{2}{3}}$ | (f) $27^{\frac{2}{3}}$ |
| (g) $81^{\frac{1}{4}}$ | (h) $32^{\frac{1}{5}}$ | (i) $32^{\frac{2}{5}}$ |
| (j) $-16^{\frac{1}{4}}$ | (k) $-81^{\frac{1}{4}}$ | (l) $-125^{\frac{2}{3}}$ |

3. Simplify.

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|--------------------------|--------------------------|----------------------------|
| (a) $4^{-\frac{1}{2}}$ | (b) $36^{-\frac{1}{2}}$ | (c) $27^{-\frac{2}{3}}$ |
| (d) $8^{-\frac{2}{3}}$ | (e) $16^{-\frac{3}{4}}$ | (f) $32^{-\frac{2}{5}}$ |
| (g) $100^{-\frac{1}{2}}$ | (h) $-8^{-\frac{1}{3}}$ | (i) $-(-8)^{-\frac{1}{3}}$ |
| (j) $125^{-\frac{2}{3}}$ | (k) $-32^{-\frac{4}{5}}$ | (l) $(-32)^{-\frac{4}{5}}$ |

4. Simplify.

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|---|---|--|
| (a) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$ | (b) $\left(\frac{27}{125}\right)^{\frac{2}{3}}$ | (c) $\left(\frac{1}{16}\right)^{-\frac{1}{2}}$ |
| (d) $\left(\frac{27}{64}\right)^{-\frac{4}{3}}$ | (e) $\left(\frac{25x^4}{m^{16}}\right)^{\frac{1}{2}}$ | (f) $\left(\frac{9a^4}{b^8}\right)$ |
| (g) $b^{\frac{1}{4}} \times b^{\frac{1}{2}}$ | (h) $a^{\frac{2}{3}} \times a^{\frac{1}{2}}$ | (i) $b^{\frac{1}{2}} \div b^{\frac{1}{4}}$ |

5. Simplify. Show only positive exponents in the solution.

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|---------------------------------------|---|--|
| (a) $\frac{a^{-6}}{a^{-7}}$ | (b) $b^{-3} \times b^{-7}$ | (c) $(x^{-2})^4$ |
| (d) $\left(\frac{x^2}{y}\right)^{-1}$ | (e) $(a^{-2}b^{-1})^{-1}$ | (f) $b^{-5} \div b^2$ |
| (g) $\frac{4a^{-2}b}{8a^{-3}b}$ | (h) $\frac{9x^{-3}y^5}{12x^{-4}y^{-5}}$ | (i) $\left(\frac{x^{-2}}{y^3}\right)^{-3}$ |
| (j) $\frac{2^0 + 3}{1 - 2^{-1}}$ | (k) $\frac{3 + 5^0}{1 - 3^{-1}}$ | (l) $\frac{1}{2^{-2}} + \frac{1}{3^{-1}}$ |

EXERCISE 1

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|--------------------|----------------------|--------------------------|-----------------------------|-----------------------------|-----------------------|
| 1. (a) 81 | (b) 32 | (c) 25 | (d) -8 | (e) -1 | (f) 36 |
| (g) 36 | (h) 16 | (i) -16 | | | |
| 2. (a) x^{11} | (b) b | (c) m^6 | (d) s^7 | (e) t^9 | (f) r^{10} |
| (g) b^9 | (h) m^{20} | (i) r^2 | (j) a^4b^6 | (k) $\frac{a^{12}}{b^{14}}$ | (l) $\frac{m^9}{n^6}$ |
| 3. (a) $12m^8$ | (b) $-21x^6$ | (c) $24t^6$ | (d) $56a^5$ | (e) $-4b^2$ | (f) $-2s^4$ |
| (g) $36x^4y^6$ | (h) $8a^6b^3c^{12}$ | (i) $\frac{16m^4}{9n^6}$ | (j) $\frac{8x^6y^3}{27z^9}$ | | |
| 4. (a) $20x^6y^4$ | (b) $18m^4n^7$ | (c) $4ab^2$ | (d) $-5x^2z$ | (e) $-42x^5y^6$ | |
| 5. (a) $-a^6b^4$ | (b) $-3r^4s^5t^{12}$ | (c) $9xyz^3$ | (d) $2a^3b^3$ | | |
| 6. (a) 3^{a+b+c} | (b) 4^v | | | | |

EXERCISE 1.5

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|-----------------------|--------------------------|-----------------------|----------------------|------------------------|------------------------|
| 1. (a) 1 | (b) 2 | (c) $\frac{1}{4}$ | (d) 16 | (e) 1 | (f) $\frac{8}{9}$ |
| (g) $\frac{1}{10000}$ | (h) 1 | (i) -1 | (j) -8 | (k) $\frac{1}{9}$ | (l) -1 |
| 2. (a) 3 | (b) 4 | (c) 2 | (d) 2 | (e) 4 | (f) 9 |
| (g) 27 | (h) 2 | (i) 8 | (j) -2 | (k) -3 | (l) -25 |
| 3. (a) $\frac{1}{2}$ | (b) $\frac{1}{6}$ | (c) $\frac{1}{9}$ | (d) $\frac{1}{4}$ | (e) $\frac{1}{8}$ | (f) $\frac{1}{4}$ |
| (g) $\frac{1}{10}$ | (h) $-\frac{1}{2}$ | (i) $\frac{1}{2}$ | (j) $\frac{1}{25}$ | (k) $-\frac{1}{16}$ | (l) $\frac{1}{16}$ |
| 4. (a) $\frac{8}{27}$ | (b) $\frac{9}{25}$ | (c) 4 | (d) $\frac{256}{81}$ | (e) $\frac{5x^2}{m^6}$ | (f) $\frac{3a^2}{b^4}$ |
| (g) b^3 | (h) $a^{\frac{7}{6}}$ | (i) $b^{\frac{1}{4}}$ | | | |
| 5. (a) a | (b) $\frac{1}{b^{10}}$ | (c) $\frac{1}{x^8}$ | (d) $\frac{y}{x^2}$ | (e) a^2b | (f) $\frac{1}{b^7}$ |
| (g) $\frac{a}{2}$ | (h) $\frac{3xy^{10}}{4}$ | (i) x^6y^9 | (j) 8 | (k) 6 | (l) 7 |