Review for test

Be able to fill in the following table

Parabola	Zeros	Vertex	Axis of Symmetry	Direction of Opening	Step Pattern	y intercept	Transformations from $y = x^2$
$y = -x^2 + 4$							
$y = 2(x-1)^2 + 8$							
y = 3(x - 4)(x + 2)							

Short Answer

1. State the coordinates of the vertex for the graph of $y = 4(x - 7)(x + 3)$.

2. A demand function is given by $p = -x + 9$, where p is the price of each item and x is the number of items sold in thousands. State the revenue function.

3. A demand function is given by $p = -3x + 19$, where p is the price in dollars and x is the number of items sold in thousands. The cost function is $C(x) = 2x + 5$. State the corresponding
   (a) revenue function
   (b) profit function

4. The profit function for a certain product is given by $P(x) = -2(x - 3)(x - 8)$, where x is the number sold in thousands. How many items must be sold for the company to break even?

5. State the quadratic function in factored form that models the following problem:
   A farmer has data indicating that if he plants 30 orange trees per acre, each tree will yield an average of 360 oranges. For every additional tree planted, 15 fewer oranges will be produced per tree. How many trees per acre should the farmer plant to maximize his total crop?
   Note: Do not solve the equation.

6. The parabola $y = 3x^2$ is translated so that the new vertex is (–2, 5). Find the corresponding quadratic function.

7. Find the value of x that will produce the maximum or minimum value of the function $g(x) = 3(x - 12)(x + 3)$. State if the function has a maximum or minimum.

8. State the number of zeros for the function $h(x) = \frac{3}{8}(x + 6)^2 + \frac{1}{2}$.

9. Given $2x^2 - 5x + 2 = 0$, state the value of the discriminant and the number and nature of the roots.

10. The y-value of the vertex of a parabola is negative and the parabola opens up. How many zeros does the corresponding function have?

11. A quadratic equation $3x^2 + bx + c = 0$ has two distinct real roots at $x = -1$ and $x = 2$. Determine the values of b and c.

Problem

12. The demand function for a new product is $p(x) = -7x + 65$, where p is the price in dollars and x is the number of items sold in thousands. The cost function is $C(x) = 9x + 105$.
    (a) Determine the profit function.
    (b) Complete the square to determine what quantity of items sold will produce the maximum profit.
    (c) Find the break-even quantities.
    (d) Determine the maximum revenue.

13. The lifeguard at a public beach has 600 m of rope available to create a rectangular swimming area. The shoreline will form one side of the rectangle. Determine the dimensions of the rectangle that will produce the largest swimming area. State what this area will be.

14. Explain the relationship between the discriminant and the number of times the graph of a quadratic function crosses the x-axis.

15. A missile is fired with an initial velocity of 250 m/s. Its height at time t in seconds after firing is given by $h(t) = -4.9t^2 + 250t$, where h is measured in metres. Is it possible for the missile to reach a height of:
    (a) 3.1 km?
    (b) 4.5 km?

16. State the form of each given function, explain the corresponding method to determine the number of zeros, then solve the problem.
    (a) $f(x) = 4x^2 - 7x + 2$

(b) $g(x) = 9.5(x-1)^2 + 1.8$

(c) $h(x) = -2(x+7)\left(x - \frac{3}{2}\right)$

17. The height of an object thrown vertically upward is modelled by the quadratic function $h(t) = s_0 + v_0 t - \frac{1}{2}gt^2$, where s_0 is the initial height above the ground, v_0 is the initial velocity, *g* is the acceleration due to gravity, *t* is the time, and *h*(*t*) is the height above the ground at time *t*.
David is playing on the balcony of his third storey apartment, 15 m above ground level, and accidently drops a tennis ball onto the grass below. The gardener picks up the ball and throws it back up to David. What initial velocity would be required for the gardener to successfully throw the ball to David? The acceleration due to gravity is 9.8 m/s^2.

Review for test
Answer Section

SHORT ANSWER

1. ANS:
(2, –100)

2. ANS:
$R(x) = x(-x + 9)$

3. ANS:
(a) $R(x) = -3x^2 + 19x$
(b) $P(x) = -3x^2 + 17x - 5$

4. ANS:
3000 or 8000

5. ANS:
$C(x) = (30 + x)(360 - 15x)$

6. ANS:
$y = 3(x+2)^2 + 5$

7. ANS:
$x = 4.5$, minimum

8. ANS:
no zeros

9. ANS:
$D = 9$, two distinct real roots

10. ANS:
two

11. ANS:
$b = -3$, $c = -6$

PROBLEM

12. ANS:

(a)

$$\begin{aligned} P(x) &= xp(x) - C(x) \\ &= x(-7x + 65) - (9x + 105) \\ &= -7x^2 + 56x - 105 \end{aligned}$$

(b)

$$\begin{aligned} P(x) &= -7\left(x^2 - 8x\right) - 105 \\ &= -7\left(x^2 - 8x + 16 - 16\right) - 105 \\ &= -7(x-4)^2 + 7 \end{aligned}$$

The vertex is (4, 7), so 4000 items must be sold to produce the maximum profit.

(c) $P(x) = -7\left(x^2 - 8x + 15\right)$

$= -7(x-3)(x-5)$

Therefore, $x = 3$ or $x = 5$, so the break-even quantities that must be sold are 3000 or 5000 items.

(d) $R(x) = xp(x)$

$= x(-7x + 65)$

The x-intercepts are $x = 0$ or $x = \frac{65}{7}$, and the midpoint value is $x \approx 4.643$.

$R(4.643) \approx 150.89$, so the maximum revenue is \$150 890.

13. ANS:
Let w represent the width of the rectangle and let l represent the length in metres.

$2w + l = 600$

$l = 600 - 2w$

$A = lw$

$A = (600 - 2w)w$

$A = -2w^2 + 600w$

$A = -2(w^2 - 300w + 22\,500 - 22\,500)$

$A = -2(w - 150)^2 + 45\,000$

When $w = 150$, $l = 300$. Thus, the dimensions of the rectangular swimming area should be $150\text{ m} \times 300\text{ m}$ and the maximum swimming area is $45\,000\text{ m}^2$.

14. ANS:
The discriminant $b^2 - 4ac$ determines the number of roots of a quadratic equation. These roots are the zeros of a quadratic function and represent the x-intercepts for the graph. When $b^2 - 4ac < 0$, there are no real roots (no zeros or x-intercepts) and the graph does not cross the x-axis. When $b^2 - 4ac = 0$, there is one double real root (one zero and one x-intercept) and the graph touches the x-axis at the vertex. When $b^2 - 4ac > 0$, there are two real roots (two zeros and two x-intercepts) and the graph crosses the x-axis at two points.

15. ANS:

(a) $-4.9t^2 + 250t - 3100 = 0$

$(250)^2 - 4(-4.9)(-3100) = 1740 > 0$

Yes, it is possible to reach a height of 3.1 km.

(b) $-4.9t^2 + 250t - 4500 = 0$

$(250)^2 - 4(-4.9)(-4500) = -25\,700 < 0$

No, it is not possible to reach a height of 4.5 km.

16. ANS:

(a) $f(x)$ is in standard form. Use the discriminant $D = b^2 - 4ac$.

$(-7)^2 - 4(4)(2) = 49 - 32 = 17 > 0$

There are two real distinct roots.

(b) $g(x)$ is in vertex form. Compare a and k values: $a = 9.5$, $k = 1.8$

Since they both have the same sign, there are no zeros.

(c) $h(x)$ is in factored form. The zeros are clearly visible at $x = -7,\ x = \frac{3}{2}$.

There are two distinct real roots.

17. ANS:
Let k represent the required initial velocity, $s_0 = 0$, and $g = 9.8$. Then $h(t) = kt - 4.9t^2$.

We need $h(t) = 15$. Therefore, $0 = -4.9t^2 + kt - 15$.

We need $b^2 - 4ac = 0$. Therefore, $k^2 - 294 = 0$.

Using $k > 0$ for upward velocity, $k \approx 17.1$.

Therefore, the gardener needs to throw the ball with an initial velocity of 17.1 m/s.